
Quantum Numbers as Closure Invariants

Discrete Labels from Exact Phase Closure

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The author developed this work through iterative conceptual exploration, formalization, and critical refinement. AI-assisted tools were used for reasoning support and editorial refinement. All conceptual direction, theoretical decisions, and responsibility for the content rest solely with the author.

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Abstract. This paper presents a structural reinterpretation of discrete quantum numbers as invariants arising from exact intrinsic phase closure. Integer labels emerge from closure conditions on compact phase descriptions without invoking operator quantization, physical spectra, or measurement postulates.

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Abstract

Quantum theory assigns discrete labels – quantum numbers – to physical states, typically introduced through operator spectra, boundary conditions, or representation theory. While mathematically effective, these constructions explain discreteness within established operator, boundary-condition, and representation-theoretic frameworks. The question addressed here is not to replace those explanations, but to offer a complementary **structural lens** that unifies integer labeling across domains.

This paper proposes a **closure-first reinterpretation**: discrete quantum numbers arise as **topological invariants of exactly closed phase evolution** on a compact manifold. Within the ResonanT⁴ framework, admissible system states are defined by exact closure of intrinsic phase states on a compact phase manifold, with an additional transverse phase governing geometric realization. The integers labeling this return – the winding vector – define equivalence classes of closure-stable trajectories. These integers function as quantum numbers in a structural sense.

It establishes a **general mechanism for discreteness** grounded in closure compatibility, supported by minimal numerical demonstrations. Discreteness is shown to emerge from closure requirements alone, without invoking quantization postulates, operators, or measurement axioms.

1. Introduction & Motivation

Discreteness is a defining feature of quantum theory. Energy levels, angular momentum, and other observables appear in discrete sets, commonly attributed to boundary conditions, commutation relations, or representation theory. These explanations specify *how* discreteness is implemented mathematically, but leave open a deeper structural question: why physically admissible states are discrete at all.

The ResonanT⁴ framework [6] adopts a different starting point: **existence requires closure**. A configuration is physically meaningful only if its internal phase evolution returns exactly, rather than drifting indefinitely. From this perspective, discreteness is not imposed – it is unavoidable.

This paper provides a **structural unification framework** for quantum numbers within a closure-compatible approach, showing how integer labeling naturally arises from exact closure on compact phase manifolds.

2. Closure as the Origin of Discreteness

Consider a system whose internal state is described by phase coordinates

$$\boldsymbol{\theta}(t) = (\theta_1(t), \dots, \theta_d(t)) \in \mathbb{T}^d.$$

A trajectory is **closure-compatible** if, after some finite evolution,

$$\Delta\boldsymbol{\theta} \in 2\pi\mathbb{Z}^d.$$

This condition is purely topological and arithmetic. It does not depend on forces, Hamiltonians, or operators. If closure fails, the trajectory never repeats and therefore carries no integer label.

The weaker word is **deliberate**. Closure is what makes a trajectory **labelable**, not what makes it robust. Whether an exactly closing trajectory survives perturbation is a separate question whose answer depends on the class of evolution applied, and in the conservative setting relevant to this paper the answer runs opposite to the naive reading. This is stated precisely in §7.1 and should be read before the closure condition is interpreted as a stability criterion.

Exact closure partitions phase space into discrete equivalence classes labeled by integers. Discreteness therefore follows from topology rather than from quantization postulates.

3. Phase Manifolds and Closure Classes

3.1 Phase Manifold

Within ResonanT⁴, intrinsic phase states form a closed modular orbit on a compact three-torus, while an additional transverse phase parametrizes geometric realization. The four coordinates are abstract phase variables and do not correspond directly to spacetime dimensions.

Compactness is essential: without it, closure has no meaning.

3.2 Closure Condition

Let the net phase advance over a full cycle be

$$\Delta\boldsymbol{\theta} = (\Delta\theta_1, \Delta\theta_2, \Delta\theta_3).$$

A configuration is admissible iff

$$\Delta\boldsymbol{\theta} = 2\pi\mathbf{n}, \quad \mathbf{n} \in \mathbb{Z}^3.$$

3.3 Closure Classes

Two trajectories are physically equivalent if they share the same winding vector

n = (n1, n2, n3).

This vector is a topological invariant in the following precise sense: it labels homotopy classes of closed loops on the intrinsic phase torus. Continuous deformations within the same homotopy class that preserve exact closure cannot change it. Closure classes are therefore discrete and structurally stable.

4. Quantum Numbers Reinterpreted

Quantum-Theoretic Concept	Closure Interpretation
Quantum number	Integer winding invariant
Discrete spectrum	Allowed closure classes
Selection rules	Closure compatibility constraints
Forbidden states	Non-closable trajectories
Degeneracy	Multiple geometries in same class
Measurement collapse	(Interpretive metaphor) projection onto nearest admissible closure class

No claim is made that specific physical quantum numbers (spin, color, flavor) are identified. The claim is structural: only integer closure is admissible, and integers therefore label admissible closure classes. They label which configurations carry a discrete invariant, not which configurations are dynamically robust; see §7.1.

5. Numerical Closure Demonstrations

These demonstrations are minimal, reproducible, and intended to illustrate the closure → discreteness mechanism. They are not physical simulations.

5.1 Toy Demonstration: Closure on T^2

Define phase evolution

theta_1(t) = omega_1 t, theta_2(t) = omega_2 t (mod 2pi).

Let the ratio r = omega_2 / omega_1 be swept across rational and irrational values.

Compute the unwrapped phase advance Delta theta and define the closure deficit

C = ||wrap(Delta theta)||.

Integer winding estimates are

$$\hat{n}_i = \text{round}(\Delta\theta_i/2\pi).$$

Exact closure occurs only for rational ratios, producing sharp minima in C and stable integer pairs (n_1, n_2) . Irrational ratios never close.

This establishes discreteness without quantization assumptions.

5.2 Resonant⁴ Closure Classes

Using three intrinsic phase coordinates, track unwrapped phase evolution over a full index cycle. Compute the winding vector $\mathbf{n} \in \mathbb{Z}^3$ and the closure deficit C .

Sweep a control parameter (e.g., detuning). Observed behavior:

- * Piecewise-constant plateaus of integer $\mathbf{n}$
- * Sharp transitions at closure boundaries
- * High-deficit regions with no stable label

These plateaus are **closure-class invariants**, analogous to discrete quantum numbers.

5.3 Operational Link via Phase Entrainment

Using event-phase data, compute phase-locking value (PLV) [3] and define operational closure

$$C_{\text{phase}} = 1 - \text{PLV}.$$

Compare resonant, off-resonant, and broadband forcing:

- * Resonant forcing stabilizes a single integer winding
- * Off-resonant forcing produces inconsistent integers
- * Broadband forcing destroys closure

This links closure invariants to observable phase-locking phenomena.

6. Required Definitions

- * **Closure-compatible resonance (CCR)**: Resonance preserving exact intrinsic phase closure.
- * **Closure deficit (C)**: Minimal distance to exact closure.

- * **Closure class:** Equivalence class of trajectories with identical winding vector.
- * **Operational closure:** $C_{\text{phase}} = 1 - \text{PLV}$.
- * **Admissibility:** Membership in a closure-compatible class.
- * **PAC:** Phase-aligned control biasing systems toward admissible closure.
- * **False resonance:** Apparent coherence without closure stability.

7. Implications & Limits

Supported

- * Discreteness arises from closure requirements.
- * Quantum numbers are topological invariants.
- * Selection rules reflect closure compatibility.

Not supported

- * That exactly closing configurations are the ones that persist under perturbation. See §7.1.
- * That closure alone produces interference structure. Closure fixes the support and the invariant label; nodes require complex amplitudes and a Maslov index supplied as an additional postulate, not derived from it.

7.1 Regime dependence of the closure criterion

This paper works with winding invariants on compact phase manifolds, which is the setting of conservative dynamics. In that setting the relationship between exact closure and survival under perturbation runs **opposite** to the direction a closure-first reading would suggest, and the point is important enough to state plainly.

KAM theory establishes that under perturbation of an integrable Hamiltonian, resonant tori are destroyed first. Survival requires a Diophantine condition on the frequency vector $[8,9]$, which is the statement that the closure deficit is bounded away from zero. The last invariant circle to break is the one with the worst rational approximation $[10,11]$. Writing the closure deficit with denominator budget N ,

$$C_N(\omega) = \min_{1 \leq n \leq N} \|n\omega\|,$$

we have $C_N = 0$ exactly for rationals of denominator at most N , while badly approximable irrationals satisfy $C_N > c/N$, with the bound maximised by the golden ratio (Hurwitz). The admissibility criterion of this framework and the KAM survival condition are therefore the **same quantity with opposite sign of preference**. That much is provable and is not in dispute.

The consequence for this paper is a scope restriction, not a retraction. Closure remains the correct criterion for **discreteness**: it is what makes an integer label exist at all, and nothing above depends on the labelled configuration being robust. What does not follow, and is not claimed here, is that the closure classes enumerated in §3 are dynamically preferred. In a conservative system they are the fragile ones.

A related caution applies to the semiclassical reading. In the hydrogen family, the configuration closest to exact closure, the circular orbit $l = n - 1$ whose frequency ratios are all rational and which closes after a single circuit, is the one case where the classical torus average does not converge to the quantum density as n grows: the quantum radial width and the classical annulus width share the same $n^{3/2}$ scaling, so their ratio does not shrink. Eccentric orbits converge, at acaustic-limited $n^{-1/3}$ rate. The most closure-degenerate configuration is where the closed-orbit picture is least accurate.

Where the closure-first criterion does carry the expected sign is **driven dissipative** evolution, in which rational rotation numbers occupy locking regions of finite width and attract a neighborhood of parameters. That is the regime of the companion paper on closure-compatible resonance [7], and the reader should not transport its conclusions into the conservative setting used here, or the reverse.

8. Conclusions & Outlook

Quantum numbers can be understood as labels of closure-stable phase evolution on compact manifolds. Within ResonanT⁴, exact closure of intrinsic phase states produces integer invariants naturally, without invoking operator quantization.

> **What is labelled is what closes.**

The stronger formulation, that what is conserved is what cannot be unclosed, overstates the result and is withdrawn. Conservation of the winding integer under deformations that preserve closure is established in §3.3; survival of the closed configuration itself under arbitrary perturbation is not, and in the conservative regime it is false (§7.1).

Future work may explore mappings between specific closure invariants and known quantum numbers, but such identification is deliberately beyond the scope of this paper.

Revision history

v1.1.0. Corrections following internal review. No numerical result or closure classification is changed; the corrections concern what the closure criterion is claimed to establish.

- §2 and §4. Closure is what makes a trajectory carry an integer label, not what makes it robust. Statements asserting that closure implies persistence are withdrawn and replaced with the labelling claim the paper actually establishes.
- Added §7.1, stating the regime dependence of the closure criterion. In the conservative setting this paper works in, KAM theory gives exact closure the opposite sign: resonant tori are destroyed first and survival requires a Diophantine condition. Added a corresponding "Not supported" list to §7 and references [8–11].
- §8. The epigraph "What is conserved is what cannot be unclosed" overstated the result and is replaced. The winding integer is conserved under closure-preserving deformation; the closed configuration itself is not guaranteed to survive arbitrary perturbation, and in this regime it does not.
- Typographic fixes in §1: two pairs of parenthetical dashes had been flattened to hyphens, joining words as "labels-quantum", "numbers-to", "return-the", "vector-define" and "imposed-it". Wording is unchanged.
- Paper I is now cited in §1 where the framework is introduced, and the PLV definition in §5.3 cites its source. Both were referred to without a corresponding citation in v1.0.0.

v1.0.0. Initial preprint.

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